Performance of Single-Relay Cooperative ARQ Retransmission Strategies

Juan J. Alcaraz and Joan García-Haro

Abstract—**In Cooperative Automatic Repeat reQuest (C-ARQ) protocols, the retransmission process between a pair of nodes can be assisted by a relay node. We investigate the performance of C-ARQ algorithms in cellular access networks, where the use of relays is a promising strategy for future evolutions. By means of Markov analysis and simulation we show that the implementation of acknowledgment signals from the relays and the retransmission policy at the base station have a notable impact on the throughput of the system.**

Index Terms—**Automatic Repeat reQuest (ARQ), cooperative diversity, multi-objective optimization.**

I. Introduction

THE use of cooperative relays in future cellular access networks is considered a promising approach to enable the deployment of 4G technology [1]. Because of their suitability in decode-and-forward relaying, C-ARQ protocols are receiving increasing attention. In this letter we study C-ARQ retransmission strategies in a time-slotted radio interface, where signals are transmitted in orthogonal channels, thus simultaneous transmissions neither collide nor interfere. The frame sending process follows a stop-and-wait operation. In a time-slot, one frame can be transmitted per channel. If a frame sent at time-slot k is successfully received, its ACK arrives, without errors, at the end of time-slot k. This scenario is similar to [2], [3].

In C-ARQ, if the relay overhears a frame that the destination node was unable to decode, it may retransmit the same copy of the frame in subsequent time-slots. As explained in [4], the throughput is maximized when both the sender and the relay retransmit the frame. However, we argue in this letter that, in a relay-enhanced cell with multiple users, the overall cell throughput increases if the base station releases some of the channels devoted to retransmissions and use them to send data to users expecting new frames.

The system is first analyzed considering a single isolated destination (D). The results show that it is possible to reduce the retransmission rate of the sender (S) with a little impact on the throughput, especially if the relay (R) can send ACK signals to the sender after correctly decoding a frame. In the multiple user case, where each user is assisted by one relay, simulation results confirm that a lower retransmission rate at the base station generates more opportunities to send new frames, increasing the throughput of the system.

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II. Model for a Single User in Isolation

In the C-ARQ protocol studied, when S sends a new data frame (n-th frame), three cases are possible. 1) If the frame is correctly decoded at D, this node ACKs frame n and frame $n+1$ is transmitted in the following time-slot. 2) If the frame is correctly decoded at R but not at D, R stores the frame. In the following time-slot both R and S may retransmit frame n, with probabilities p_R and p_S respectively. 3) If neither D nor R are able to correctly decode the frame, then S may retransmit with probability p_S^0. The system persists until correct frame reception. Two different options are studied. First, if the system does not include a mechanism to inform S of a correct frame reception at R (ACK-R), then $p_S^0 = p_S$. Second, if the ACK-R mechanism is implemented, then $p_S^0 = 1$. The following analysis allows us to configure p_S in both cases.

The frame transmission process is modeled with a Discrete-Time Markov Chain (DTMC) with three states: frame reception error at both D and R (state 0), correct frame reception at R but not at D (state 1), and correct frame reception at D (state 2). The transition probabilities are determined by the Frame Error Ratio (FER) of each channel. The FER of the channel between S and D (*direct link*) is denoted by p_{SD}. p_{SR} is the FER of the *relay link* (S - R) and p_{RD} corresponds to the *access link* (R - D). Once the system enters state 2, a new frame is transmitted, and the system can make a transition to any state. The relay is assumed not to discard frames, therefore transitions from state 1 to state 0 are forbidden ($p_{10} = 0$). It can be easily checked that the rest of the transition probabilities are:

$$\begin{aligned}
p_{00} &= p_S^0 p_{SD} p_{SR} + (1 - p_S^0) \\
p_{01} &= p_S^0 p_{SD}(1 - p_{SR}) \\
p_{02} &= p_S^0 (1 - p_{SD}) \\
p_{11} &= (p_R p_{RD} + (1 - p_R))(p_S p_{SD} + (1 - p_S)) \\
p_{12} &= 1 - p_{11} \\
p_{20} &= p_{SD} p_{SR} \\
p_{21} &= p_{SD}(1 - p_{SR}) \\
p_{22} &= 1 - p_{SD}
\end{aligned} \tag{1}$$

where $p_S^0 = 1$ with ACK-R and $p_S^0 = p_S$ otherwise. Let $\overline{\pi} = \{\pi_0, \pi_1, \pi_2\}$ be the steady-state distribution of the DTMC. $\overline{\pi}$ is obtained by solving the following system of linear equations:

$$\begin{gathered}
\overline{\pi} = \overline{\pi}\mathbf{P} \\
\sum_{i \in \Omega} \pi_i = 1
\end{gathered} \tag{2}$$

where $\Omega = \{0, 1, 2\}$ and $\mathbf{P}$ is the transition matrix whose elements are given by (1).

The throughput, defined as the average number of frames successfully received at the destination node per time-slot, can be computed as the average number of time-slots that the DTMC spends in state 2.

The retransmission rate at S is defined as the ratio between the number of retransmissions from S and the total number of time-slots. This metric is obtained by means of a reward model associated with the DTMC. This approach has been previously applied to the analysis of ARQ protocols in [5].

Considering a generic transition $i \rightarrow j$, let N_{ij} denote the retransmission rate at S related to this transition (associated reward). N_{ij} can be computed as the fraction of transitions $i \rightarrow j$ for which a retransmission event at S (S_{rt}) takes place. Analytically, it is expressed as $N_{ij} = P\{S_{rt}|i \rightarrow j\}$. The retransmission rate associated to a reference state, i, is computed with the following expression:

$$N_i = \sum_{j \in \Omega} p_{ij} N_{ij}. \tag{3}$$

By applying the law of total probability, the resulting rates for the system without ACK-R are $N_0 = N_1 = p_S$ and $N_2 = 0$. Observe that, if ACK-R is used, $N_0 = 1$. The average retransmission rate (N) is given by a weighted sum of the rates in (3), where the weighting factors are the steady state probabilities of the Markov chain:

$$N = \sum_{i \in \Omega} \pi_i N_i. \tag{4}$$

In order to focus on adjusting p_S, we express, for convenience, the steady state distribution of the system as $\overline{\pi}(p_S)$. Then, the throughput is given by $T(p_S) = \pi_2(p_S)$. Because the maximum throughput is achieved when $p_S = 1$, we define the maximum throughput as $T_M = T(1)$. T_M is taken as a reference for performance evaluation. Similarly, we can express N in terms of p_S as $N(p_S) = p_S(1 - \pi_2(p_S))$, for a system without ACK-R, and $N(p_S) = \pi_0(p_S) + p_S \pi_1(p_S)$ if ACK-R is used. The retransmission rate associated with T_M is $N_M = N(1)$.

III. Multi-Objective Optimization Strategy

In the single user case, we are interested in finding a value for p_S that maximizes $T(p_S)$ but also that minimizes $N(p_S)$, because, in the multiple user scenario, it will provide S with more transmission opportunities for new frames. The maximum throughput, T_M, is achieved by setting $p_S = 1$. However, this can lead to a high retransmission rate, especially if the direct link is highly degraded. On the other hand, if $p_S = 0$, then $N(p_S)$ reduces to 0. A feasible approach to balance both opposed objectives is the global criterion method, a multi-objective optimization technique that consists in minimizing a global criterion function, defined as:

$$G(p_S) = \sum_{k=1}^{n} \alpha_k \left(\frac{O_k - f_k(p_S)}{O_k} \right)^2 \tag{5}$$

where n is the number of objective functions, $f_k(p_S)$ are the objective functions, O_k are the optimum values for each objective function and α_k are weighting factors for the relative importance of each objective. In the system analyzed, we are balancing two objectives. First, it is desirable to obtain a throughput as high as possible, i.e. $f_1(p_s) = T(p_s)$ and $O_1 = T_M$. Second, it is also desirable to reduce N, therefore $f_2(p_s) = N(p_s) + C$, and $O_2 = C$, where C is an auxiliary non-zero real number required to avoid a division by 0 in the global objective, that, for the sake of simplicity is set to $C = 1$. Applying these definitions in (5) we obtain:

$$G(p_S) = \alpha_1 \left(\frac{T_M - T(p_S)}{T_M} \right)^2 + \alpha_2 N(p_S)^2. \tag{6}$$

Let p_S^* denote the solution to the multi-objective optimization problem. Because p_S^* is a probability, we must consider the constraint $0 \leq p_S \leq 1$. Therefore we have:

$$p_S^* = \underset{0 \leq p_S \leq 1}{\arg\min} \{G(p_S)\}. \tag{7}$$

Let $T_O = T(p_S^*)$ be the optimum throughput in terms of the multi-objective optimization problem. Similarly, let $N_O = N(p_S^*)$ denote the optimum retransmission rate. In the following section we compare T_O and N_O with T_M and N_M for a system with ACK-R and a system without it. The weighting factors are set to $\alpha_1 = 1$ and $\alpha_2 = 0.2$, assigning more importance to the throughput in the single user case.

IV. Numerical Results

In this section we show the performance results in a typical relay enhanced cell scenario. We begin with the single isolated user model described in previous sections. Considering a typical scenario, where a relay is associated to a destination when they are close to each other and relays are deployed to have relatively good coverage from the base station, the access and relay links have relatively good propagation conditions, and thus their FERs are set to $p_{RD} = 0.1$ and $p_{SR} = 0.2$ respectively. Relays are assumed to be deployed as part of the radio cell infrastructure and therefore to be fully devoted to cooperation. In consequence $p_R = 1$. Fig. 1 plots the performance metrics versus p_{SD} in a system without ACK-R. It is interesting to observe that, as the FER of the direct link increases, N_O is notably reduced compared to N_M, while T_O remains relatively close to T_M. The value of p_S^*, not shown in the figure, lays between 0.6 and 0.5 in this scenario. Fig. 2 depicts the same measures for an ACK-R system. In this system, $p_S^* = 0$ for all values of p_{SD}, resulting in an even more reduced N_O and, what is more interesting, with an almost negligible effect on the throughput.

By means of simulation it is verified that the reduction in N obtained with ACK-R in the single user case implies an increment of the cell throughput in the multiple user case. The simulated scenario consists of a relay-enhanced cell with 20 radio channels serving 30 users. 10 channels are used by the base station and the remaining 10 channels are used for relay retransmissions. Channels are assigned to each link in a round-robin, centralized fashion. Each channel allows the transmission of a single frame each time-slot. Every user is relay-assisted and FERs are configured as in the single user case previously discussed. The basic idea in the multiple user

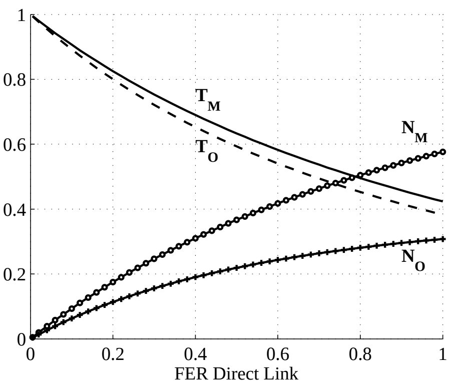

Fig. 1. Performance vs. p_{SD} without ACK-R.

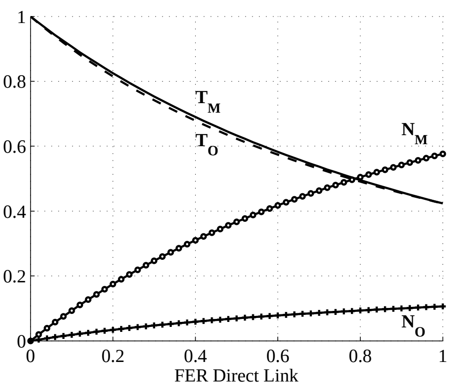

Fig. 2. Performance vs. p_{SD} with ACK-R.

system is to take advantage of the retransmission rate reduction by using the channels that remain idle in the single user case to introduce new frames into the cell, if there are users that can receive a new frame. Let $A(k)$ denote the set of users that have ACKed every frame sent to them up to time-slot $k-1$. Note that, in time-slot k, new frames can only be sent to destinations belonging to $A(k)$, due to the stop-and-wait operation of the algorithm, which also implies that a relay only needs to store up to one frame per assigned user.

Let us consider that, in time-slot k, the base station has to retransmit frame n directed to user D_i, which direct link is associated to channel c. Assume that R stores frame n (R has ACKed frame n). If $A(k) = \emptyset$, S retransmits frame n. If $A(k) \neq \emptyset$, S retransmits frame n with probability p_S or uses channel c to send a new frame to another user, $D_j \in A(k)$, with probability $(1 - p_S)$. If D_i ACKs frame n at the end of time-slot k, then for time-slot $k+1$, channel c is assigned to the direct link of the next user (D_m) in the round-robin turn such that $D_m \in A(k+1)$. If not, S decides again whether to retransmit frame n or not. Regarding R, it will retransmit frame n until D_i ACKs the frame, whenever there is a channel assigned to the access link between R and D_i. The duration of time-slots is set to 10 ms. Simulation results have a confidence interval of 5% or better, at 95% confidence level.

Fig. 3 (a) plots the throughput per user, defined as the number of acknowledged frames per second. These measures are shown for different values of p_S. As expected, the retrans-

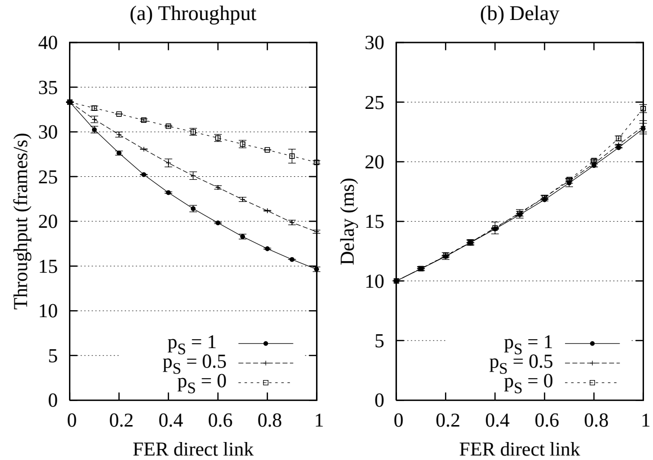

Fig. 3. Performance vs. p_{SD} in a cell with 30 users, 10 channels for the base station and 10 channels for the relays.

mission probability yielding the lowest retransmission rate in the single user model ($p_S = 0$), attains the highest throughput in the multiple user scenario. Fig. 3 (b) depicts the average delay per packet, defined as the time elapsed between the first transmission of a frame and the reception of its ACK. Observe that the delay, which is directly related to the throughput computed for an isolated user, is not noticeable degraded by the reduction on the retransmission rate.

V. Conclusions

In this letter, we investigate different configurations of the C-ARQ protocol. Considering a single user in isolation, the retransmission rate of the source can be reduced with a small impact on the throughput using a probabilistic retransmission strategy at the source, adjusted by means of a multi-objective optimization algorithm. This effect is more noticeable when the relays can send ACKs. In a cell with multiple users, the channels released when the base station decides not to retransmit can be used to introduce new data in the cell. In consequence, the highest throughput is achieved when the base station fully delegates the retransmission process to the relays that have sent an ACK. Interestingly, this strategy is contrary to the classical policy of always giving maximum priority to retransmissions in wireless access nodes.

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